Mathematik * Jahrgangsstufe 9 * Gleichungen und Rechnungen mit Wurzeln

1. Löse – falls möglich – die folgenden Gleichungen!

a) $4\cdot(3-2x^2)=1$ b) $3\cdot(2x^2+1)=x^2-2$

c) $2x-5=(x+1)^2-1$ d) $2x^2-6x=(x-3)^2+3$

e) $2x\cdot(4+x)+6=4\cdot(3+2x)$ f) $(x-2)\cdot(x+2)=2x^2-4^2$

2. Löse – falls möglich – die folgenden Wurzelgleichungen! Vergiss die Probe nicht!

a) $\sqrt{5x-4}=3$ b) $6-\sqrt{5+4x}=3$

c) $2\cdot\sqrt{5+4x}=5$ d) $4+\sqrt{5-\sqrt{x}}=6$

e) $1+\sqrt{4x^2-7}=2x$ f) $\sqrt{4x^2-7}=2x-7$

g) $\sqrt{4+\sqrt{5x}}=3$ h) $1+\sqrt{5+x^2}=x+2$

i) $\sqrt{4x^2+3}=1-2x$ k) $2+\sqrt{5+x^2}=x+1$

l) $\sqrt{4x^2+3}=1+2x$ m) $2+\sqrt{5+x^2}=x+3$

3. Vereinfache den Term!
Mache also den Nenner rational und radiziere so weit wie möglich.

a) $\dfrac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$ b) $\dfrac{4+2\cdot\sqrt{8}}{\sqrt{8}-2}$

c) $\dfrac{\sqrt{12}}{\sqrt{6}}+\dfrac{\sqrt{6}}{\sqrt{12}}+\dfrac{\sqrt{5}}{\sqrt{10}}$ d) $\dfrac{3}{2+\sqrt{3}}-\dfrac{\sqrt{3}-3\cdot\sqrt{2}}{\sqrt{6}}+\dfrac{1}{\sqrt{2}}$

e) $\dfrac{(1+\sqrt{2})\cdot(3-\sqrt{2})}{\sqrt{2}-1}$ f) $\left(\dfrac{2}{\sqrt{5}}-\dfrac{\sqrt{5}}{2}\right)\cdot\dfrac{4\cdot\sqrt{5}}{2+\sqrt{5}}$

4. Bestimme den Definitionsbereich und vereinfache!

a) $\sqrt{108x^5y^2}$ b) $\dfrac{\sqrt{288a^3b^6}}{\sqrt{6a^2}}$

c) $\dfrac{\sqrt{28x^2}+2\sqrt{x}}{\sqrt{63xy^4}}$ d) $\dfrac{\sqrt{x}}{2-\sqrt{x}}+\dfrac{2\sqrt{x}}{x-4}$

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Lösungen

1. a) $4\cdot(3-2x^2)=1 \Leftrightarrow 12-8x^2=1 \Leftrightarrow 8x^2=11 \Leftrightarrow x^2=\frac{11}{8} \Leftrightarrow x_{1/2}=\pm\sqrt{\frac{11}{8}}=\pm\frac{\sqrt{22}}{4}$

   b) $3\cdot(2x^2+1)=x^2-2 \Leftrightarrow 6x^2+3=x^2-2 \Leftrightarrow 5x^2=-5 \Leftrightarrow x^2=-1$ keine Lösung!!

   c) $2x-5=(x+1)^2-1 \Leftrightarrow 2x-5=x^2+2x+1-1 \Leftrightarrow -5=x^2$ keine Lösung!!

   d) $2x^2-6x=(x-3)^2+3 \Leftrightarrow 2x^2-6x=x^2-6x+9+3 \Leftrightarrow x^2=12 \Leftrightarrow x_{1/2}=\pm2\cdot\sqrt{3}$

   e) $2x\cdot(4+x)+6=4\cdot(3+2x) \Leftrightarrow 8x+2x^2+6=12+8x \Leftrightarrow 2x^2=6 \Leftrightarrow x_{1/2}=\pm\sqrt{3}$

   f) $(x-2)\cdot(x+2)=2x^2-4^2 \Leftrightarrow x^2-4=2x^2-16 \Leftrightarrow 12=x^2 \Leftrightarrow x_{1/2}=\pm\sqrt{12}=\pm2\cdot\sqrt{3}$

2. a) $\sqrt{5x-4}=3 \Rightarrow 5x-4=9 \Rightarrow 5x=13 \Rightarrow x=\frac{13}{5}=2{,}6$

   Probe: l.S.: $\sqrt{5\cdot\frac{13}{5}-4}=\sqrt{9}=3$ und r.S.: 3 also $L=\{\frac{13}{5}\}$

   b) $6-\sqrt{5+4x}=3 \Rightarrow 3=\sqrt{5+4x} \Rightarrow 9=5+4x \Leftrightarrow x=1$

   Probe: l.S.: $6-\sqrt{5+4\cdot1}=6-\sqrt{9}=6-3=3$ r.S.: 3 also $L=\{1\}$

   c) $2\cdot\sqrt{5+4x}=5 \Rightarrow \sqrt{5+4x}=2{,}5 \Rightarrow 5+4x=6{,}25 \Rightarrow 4x=1{,}25 \Rightarrow x=\frac{5}{16}$

   Probe: l.S.: $2\cdot\sqrt{5+4\cdot\frac{5}{16}}=2\cdot\sqrt{6{,}25}=2\cdot2{,}5=5$ r.S.: 5 also $L=\{\frac{5}{16}\}$

   d) $4+\sqrt{5-\sqrt{x}}=6 \Rightarrow \sqrt{5-\sqrt{x}}=2 \Rightarrow 5-\sqrt{x}=4 \Rightarrow 1=\sqrt{x} \Rightarrow x=1$

   Probe: l.S. $4+\sqrt{5-\sqrt{1}}=4+\sqrt{4}=4+2=6$ r.S.: 6 also $L=\{1\}$

   e) $1+\sqrt{4x^2-7}=2x \Rightarrow 4x^2-7=(2x-1)^2 \Rightarrow 4x^2-7=4x^2-4x+1 \Rightarrow$
   $-7=-4x+1 \Rightarrow 4x=8 \Rightarrow x=2$

   Probe: l.S. $1+\sqrt{4\cdot2^2-7}=1+\sqrt{9}=1+3=4$ r.S.: $2\cdot2=4$ also $L=\{2\}$

   f) $\sqrt{4x^2-7}=2x-7 \Rightarrow 4x^2-7=4x^2-28x+49 \Rightarrow 28x=56 \Rightarrow x=2$

   Probe: l.S.: $\sqrt{4\cdot2^2-7}=\sqrt{9}=3$ r.S.: $2\cdot2-7=-3$ also $L=\{\ \}$

   g) $\sqrt{4+\sqrt{5x}}=3 \Rightarrow 4+\sqrt{5x}=9 \Rightarrow \sqrt{5x}=5 \Rightarrow 5x=5^2 \Rightarrow x=5$

   Probe: l.S.: $\sqrt{4+\sqrt{5\cdot5}}=\sqrt{4+5}=\sqrt{9}=3$ r.S.: 3 also $L=\{5\}$

   h) $1+\sqrt{5+x^2}=x+2 \Rightarrow 5+x^2=(x+1)^2 \Rightarrow 5+x^2=x^2+2x+1 \Rightarrow x=2$

   Probe: r.S.: $1+\sqrt{5+2^2}=1+\sqrt{9}=4$ r.S.: $2+2=4$ also $L=\{2\}$

   i) $\sqrt{4x^2+3}=1-2x \Rightarrow 4x^2+3=1-4x+4x^2 \Rightarrow 2=-4x \Rightarrow x=-0{,}5$

   Probe: l.S.: $\sqrt{4\cdot(-0{,}5)^2+3}=\sqrt{4}=2$ r.S.: $1-2\cdot(-0{,}5)=2$ also $L=\{-0{,}5\}$

k) $2+\sqrt{5+x^2} = x+1 \Rightarrow 5+x^2 = (x-1)^2 \Rightarrow 5+x^2 = x^2-2x+1 \Rightarrow 4 = -2x \Rightarrow x = -2$

Probe: l.S.: $2+\sqrt{5+(-2)^2} = 2+\sqrt{9} = 5$ r.S.: $-2+1 = -1$ also $L = \{\ \}$

l) $\sqrt{4x^2+3} = 1+2x \Rightarrow 4x^2+3 = 1+4x+4x^2 \Rightarrow 2 = 4x \Rightarrow x = 0,5$

Probe: l.S.: $\sqrt{4\cdot(0,5)^2+3} = \sqrt{4} = 2$ r.S.: $1+2\cdot 0,5 = 2$ also $L = \{0,5\}$

m) $2+\sqrt{5+x^2} = x+3 \Rightarrow 5+x^2 = (x+1)^2 \Rightarrow 5 = 2x+1 \Rightarrow x = 2$

Probe: l.S.: $2+\sqrt{5+2^2} = 2+\sqrt{9} = 5$ r.S.: $2+3 = 5$ also $L = \{2\}$

3. a) $\dfrac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} = \dfrac{\sqrt{6}\cdot(\sqrt{3}-\sqrt{2})}{(\sqrt{2}+\sqrt{3})\cdot(\sqrt{3}-\sqrt{2})} = \dfrac{\sqrt{18}-\sqrt{12}}{3-2} = 3\cdot\sqrt{2} - 2\cdot\sqrt{3}$

b) $\dfrac{4+2\cdot\sqrt{8}}{\sqrt{8}-2} = \dfrac{(4+4\cdot\sqrt{2})\cdot(\sqrt{8}+2)}{(\sqrt{8}-2)\cdot(\sqrt{8}+2)} = \dfrac{4\cdot 2\sqrt{2}+8+4\cdot\sqrt{16}+8\sqrt{2}}{8-4} = \dfrac{16\sqrt{2}+24}{4} = 6+4\sqrt{2}$

c) $\dfrac{\sqrt{12}}{\sqrt{6}} + \dfrac{\sqrt{6}}{\sqrt{12}} + \dfrac{\sqrt{5}}{\sqrt{10}} = \sqrt{2} + \sqrt{\dfrac{1\cdot 2}{2\cdot 2}} + \sqrt{\dfrac{1\cdot 2}{2\cdot 2}} = \sqrt{2} + \dfrac{1}{2}\sqrt{2} + \dfrac{1}{2}\sqrt{2} = 2\cdot\sqrt{2}$

d) $\dfrac{3}{2+\sqrt{3}} - \dfrac{\sqrt{3}-3\cdot\sqrt{2}}{\sqrt{6}} + \dfrac{1}{\sqrt{2}} = \dfrac{3\cdot(2-\sqrt{3})}{(2+\sqrt{3})\cdot(2-\sqrt{3})} - \dfrac{(\sqrt{3}-3\cdot\sqrt{2})\cdot\sqrt{6}}{\sqrt{6}\cdot\sqrt{6}} + \dfrac{\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} =$

$\dfrac{6-3\sqrt{3}}{4-3} - \dfrac{3\sqrt{2}-3\cdot 2\cdot\sqrt{3}}{6} + \dfrac{\sqrt{2}}{2} = 6-3\sqrt{3}-0,5\sqrt{2}+\sqrt{3}+0,5\sqrt{2} = 6-2\cdot\sqrt{3}$

e) $\dfrac{(1+\sqrt{2})\cdot(3-\sqrt{2})}{\sqrt{2}-1} = \dfrac{3-\sqrt{2}+3\sqrt{2}-2}{\sqrt{2}-1} = \dfrac{(1+2\sqrt{2})\cdot(\sqrt{2}+1)}{(\sqrt{2}-1)\cdot(\sqrt{2}+1)} = \dfrac{\sqrt{2}+1+2\cdot 2+2\sqrt{2}}{2-1} =$

$5+3\sqrt{2}$

f) $\left(\dfrac{2}{\sqrt{5}} - \dfrac{\sqrt{5}}{2}\right)\cdot\dfrac{4\cdot\sqrt{5}}{2+\sqrt{5}} = \left(\dfrac{2\cdot\sqrt{5}}{5} - \dfrac{\sqrt{5}}{2}\right)\cdot\dfrac{4\cdot\sqrt{5}\cdot(2-\sqrt{5})}{(2+\sqrt{5})\cdot(2-\sqrt{5})} = -\dfrac{1}{10}\sqrt{5}\cdot\dfrac{8\sqrt{5}-4\cdot 5}{4-5} =$

$-\dfrac{1}{10}\sqrt{5}\cdot(-8\sqrt{5}+4\cdot 5) = \dfrac{8}{10}\cdot\sqrt{5\cdot 5} - 2\sqrt{5} = 4-2\sqrt{5}$

4. a) $\sqrt{108x^5y^2} = \sqrt{4\cdot 3\cdot 9x^4xy^2} = 2\cdot 3\cdot x^2\cdot|y|\cdot\sqrt{3x} = 6\cdot x^2\cdot|y|\cdot\sqrt{3x}$ mit $x\in R_o^+,\ y\in R$

b) $\dfrac{\sqrt{288a^3b^6}}{\sqrt{6a^2}} = \sqrt{\dfrac{6\cdot 48\cdot a^3b^6}{6a^2}} = \sqrt{3\cdot 16\cdot a\cdot b^6} = 4|b^3|\cdot\sqrt{3a}$ mit $a\in R^+,\ b\in R$

c) $\dfrac{\sqrt{28x^2}+2\sqrt{x}}{\sqrt{63xy^4}} = \sqrt{\dfrac{4\cdot\cancel{7}\cdot\cancel{x}\cdot x}{9\cdot\cancel{7}\cdot\cancel{x}\cdot y^4}} + 2\cdot\sqrt{\dfrac{\cancel{x}\cdot 7}{9\cdot 7\cdot 7\cdot\cancel{x}\cdot y^4}} = \dfrac{2\cdot\sqrt{x}}{3y^2} + \dfrac{2\cdot\sqrt{7}}{3\cdot 7\cdot y^2} = \dfrac{14\sqrt{x}+2\sqrt{7}}{21y^2}$

mit $x\in R^+,\ y\in R\setminus\{0\}$

d) $\dfrac{\sqrt{x}}{2-\sqrt{x}} + \dfrac{2\sqrt{x}}{x-4} = \dfrac{\sqrt{x}\cdot(2+\sqrt{x})}{(2-\sqrt{x})\cdot(2+\sqrt{x})} + \dfrac{2\sqrt{x}}{x-4} = \dfrac{2\sqrt{x}+x}{4-x} - \dfrac{2\sqrt{x}}{4-x} = \dfrac{x}{4-x}$

mit $x\in R_o^+\setminus\{4\}$