Mathematik * Klasse 9c * Rechnen mit Wurzeln

Merke: Für $a \in \mathbb{R}_0^+$ legt man fest: $\sqrt{a}$ ist die <u>positive</u> Zahl, für die gilt $\left(\sqrt{a}\right)^2 = a$

Für Wurzeln gelten die beiden folgenden Regeln:

$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$ und $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$ Zudem gilt:

Die quadratische Gleichung $x^2 = a$ hat für $a > 0$ die beiden Lösungen $x_1 = \sqrt{a}$ und $x_2 = -\sqrt{a}$.

Beachte: Es muss immer so weit wie möglich radiziert und der Nenner rational gemacht werden.

Also z.B. $\sqrt{12a^3} = \sqrt{4 \cdot 3 \cdot a^2 \cdot a} = 2 \cdot a \cdot \sqrt{3a}$ und $\dfrac{2}{\sqrt{3}} = \dfrac{2 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \dfrac{2 \cdot \sqrt{3}}{3}$

Aufgaben:

1. Bestimme den Definitionsbereich des Terms!

a) $\sqrt{2x - 3}$ b) $\sqrt{0{,}4x + 2}$ c) $\sqrt{x^2 + 1}$

d) $\sqrt{4 - 0{,}2x}$ e) $\sqrt{4 - x^2}$ f) $\sqrt{x \cdot (2 - x)}$

g) $\sqrt{\dfrac{2}{x}}$ h) $\sqrt{\dfrac{3}{2 + x}}$ k) $\dfrac{\sqrt{x+1}}{1 + \sqrt{x}}$

2. Vereinfache durch Ausmultiplizieren und Zusammenfassen

a) $2\sqrt{6} \cdot \left(\sqrt{3} - 2\sqrt{2}\right) - \sqrt{10} \cdot \left(\sqrt{15} - \sqrt{30}\right)$ b) $\left(1 + \sqrt{3}\right)^2 - \left(1 - \sqrt{3}\right)^2$

c) $\left(\sqrt{6} - 2\sqrt{2}\right)^2 - \sqrt{2} \cdot \left(\sqrt{18} - \sqrt{150}\right)$ d) $\left(2 - 3\sqrt{5}\right)^2 - \left(1 + \sqrt{20}\right)^2$

e) $\sqrt{8} \cdot \left(\sqrt{6} - 2\sqrt{5}\right) + \sqrt{5} \cdot \left(\sqrt{15} - \sqrt{2}\right) + \sqrt{2} \cdot \left(6\sqrt{5} - 5\sqrt{6}\right)$

3. Bestimme die Definitionsmenge und radiziere dann so weit wie möglich. Nenner rational machen!

a) $\sqrt{250a^3b^4}$ b) $\sqrt{80x^6y^8z^5}$ c) $\sqrt{27a^3b^{50}}$

d) $\dfrac{\sqrt{8x^2y}}{\sqrt{6xy^2}}$ e) $\dfrac{\sqrt{24a^2b^3}}{\sqrt{15ab}}$ f) $\dfrac{\sqrt{1000x^{20}y}}{\sqrt{75xy^4}}$

4. Vereinfache

a) $\dfrac{\sqrt{12}}{\sqrt{15}}$ b) $\dfrac{\sqrt{8}}{\sqrt{75}}$ c) $\sqrt{12{,}5}$ d) $\sqrt{12{,}1}$ e) $\sqrt{4\frac{1}{2}} - \sqrt{3\frac{1}{8}}$

f) $\dfrac{\sqrt{3}}{1 + \sqrt{2}}$ g) $\dfrac{1}{2 - \sqrt{3}}$ h) $\dfrac{1 - \sqrt{2}}{2 + \sqrt{2}}$ i) $\dfrac{\sqrt{8}}{2 - \sqrt{2}}$ k) $\dfrac{\sqrt{6}}{\sqrt{3} - \sqrt{2}}$

m) $\dfrac{\sqrt{8} - \sqrt{6}}{\sqrt{12}}$ n) $\dfrac{\sqrt{12}}{1 + \sqrt{2}}$ p) $\dfrac{2 + \sqrt{2}}{\left(1 - \sqrt{2}\right)^2}$ q) $\dfrac{1 - \sqrt{2}}{\left(1 + \sqrt{2}\right)^2}$ r) $\dfrac{\sqrt{12}}{\left(1 - \sqrt{3}\right)^2}$

5. Bestimme alle Lösungen!

a) $2x^2 - 3 = 4$ b) $0{,}5x^2 + 2 = 3$ c) $2 \cdot (x^2 - 1) = 5$ d) $(x - 3) \cdot (x + 3) = 1$

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1. a) $D = [1{,}5\,;\,\infty[$ b) $D = [-5\,;\,\infty[$ c) $D = \mathbb{R}$
   d) $D =]-\infty\,;\,20\,]$ e) $D = [-2\,;\,2\,]$ f) $D = [\,0\,;\,2\,]$
   g) $D = \mathbb{R}^+ = \;]\,0\,;\,\infty[$ h) $D =]-2\,;\,\infty[$ k) $D = \mathbb{R}^+ = \;]\,0\,;\,\infty[$

2. a) $2\sqrt{6}\cdot\left(\sqrt{3}-2\sqrt{2}\right)-\sqrt{10}\cdot\left(\sqrt{15}-\sqrt{30}\right)=6\sqrt{2}-8\sqrt{3}-5\sqrt{6}+10\sqrt{3}=$
   $6\sqrt{2}+2\sqrt{3}-5\sqrt{6}$

   b) $\left(1+\sqrt{3}\right)^2-\left(1-\sqrt{3}\right)^2=1+2\sqrt{3}+3-\left(1-2\sqrt{3}+3\right)=4\sqrt{3}$

   c) $\left(\sqrt{6}-2\sqrt{2}\right)^2-\sqrt{2}\cdot\left(\sqrt{18}-\sqrt{150}\right)=6-4\sqrt{12}+8-6+10\sqrt{3}=8+2\sqrt{3}$

   d) $\left(2-3\sqrt{5}\right)^2-\left(1+\sqrt{20}\right)^2=4-12\sqrt{5}+45-\left(1+4\sqrt{5}+20\right)=28-16\sqrt{5}$

   e) $\sqrt{8}\cdot\left(\sqrt{6}-2\sqrt{5}\right)+\sqrt{5}\cdot\left(\sqrt{15}-\sqrt{2}\right)+\sqrt{2}\cdot\left(6\sqrt{5}-5\sqrt{6}\right)=$
   $4\sqrt{3}-4\sqrt{10}+5\sqrt{3}-\sqrt{10}+6\sqrt{10}-10\sqrt{3}=\sqrt{10}-\sqrt{3}$

3. a) $\sqrt{250a^3b^4}=5\cdot a\cdot b^2\cdot\sqrt{10a}$ mit $a\in\mathbb{R}_o^+$ und $b\in\mathbb{R}$

   b) $\sqrt{80x^6y^8z^5}=4\cdot\left|x^3\right|\cdot y^4\cdot z^2\cdot\sqrt{5z}$ mit $x\in\mathbb{R}$, $y\in\mathbb{R}$, $z\in\mathbb{R}_o^+$

   c) $\sqrt{27a^3b^{50}}=3\cdot a\cdot\left|b^{25}\right|\cdot\sqrt{3a}$ mit $a\in\mathbb{R}_o^+$, $b\in\mathbb{R}$

   d) $\dfrac{\sqrt{8x^2y}}{\sqrt{6xy^2}}=\sqrt{\dfrac{4\cdot x\cdot y\cdot 3}{3\cdot y^2\cdot 3}}=\dfrac{2\cdot\sqrt{3xy}}{3y}$ mit $x\in\mathbb{R}_o^+$, $y\in\mathbb{R}_o^+$

   e) $\dfrac{\sqrt{24a^2b^3}}{\sqrt{15ab}}=\sqrt{\dfrac{8\cdot a\cdot b^2\cdot 5}{5\cdot 5}}=\dfrac{2b\cdot\sqrt{10a}}{5}$ mit $b\in\mathbb{R}_o^+$ und $a\in\mathbb{R}_o^+$

   f) $\dfrac{\sqrt{1000x^{20}y}}{\sqrt{75xy^4}}=\sqrt{\dfrac{100\cdot 10\cdot x^{19}\cdot y\cdot 3}{25\cdot 3\cdot y^4\cdot 3}}=\dfrac{10\cdot x^9\cdot\sqrt{30xy}}{25\cdot 3\cdot y^2}=\dfrac{2x^9\cdot\sqrt{30xy}}{15y^2}$ mit $x\in\mathbb{R}_o^+$, $y\in\mathbb{R}_o^+$

4. a) $\dfrac{\sqrt{12}}{\sqrt{15}}=\sqrt{\dfrac{4}{5}}=\sqrt{\dfrac{4\cdot 5}{5\cdot 5}}=\dfrac{2\sqrt{5}}{5}$ $(=0{,}4\cdot\sqrt{5})$ b) $\dfrac{\sqrt{8}}{\sqrt{75}}=\dfrac{2\cdot\sqrt{2}\cdot\sqrt{3}}{5\cdot\sqrt{3}\cdot\sqrt{3}}=\dfrac{2\sqrt{6}}{15}$

   c) $\sqrt{12{,}5}=\sqrt{\dfrac{25}{2}}=\sqrt{\dfrac{25\cdot 2}{2\cdot 2}}=\dfrac{5\sqrt{2}}{2}$ $(=2{,}5\cdot\sqrt{2})$

   d) $\sqrt{12{,}1}=\sqrt{\dfrac{121\cdot 10}{10\cdot 10}}=\dfrac{11\sqrt{10}}{10}$ $(=1{,}1\cdot\sqrt{10})$

   e) $\sqrt{4\frac{1}{2}}-\sqrt{3\frac{1}{8}}=\sqrt{\dfrac{9\cdot 2}{2\cdot 2}}-\sqrt{\dfrac{25\cdot 2}{8\cdot 2}}=\dfrac{3\sqrt{2}}{2}-\dfrac{5\sqrt{2}}{4}=\dfrac{6\sqrt{2}-5\sqrt{2}}{4}=\dfrac{\sqrt{2}}{4}$

   f) $\dfrac{\sqrt{3}}{1+\sqrt{2}}=\dfrac{\sqrt{3}\cdot\left(1-\sqrt{2}\right)}{\left(1+\sqrt{2}\right)\cdot\left(1-\sqrt{2}\right)}=\dfrac{\sqrt{3}-\sqrt{6}}{1-2}=\sqrt{6}-\sqrt{3}$

   g) $\dfrac{1}{2-\sqrt{3}}=\dfrac{1\cdot\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)\cdot\left(2+\sqrt{3}\right)}=\dfrac{2+\sqrt{3}}{4-3}=2+\sqrt{3}$

   +

h) $\frac{1-\sqrt{2}}{2+\sqrt{2}} = \frac{\left(1-\sqrt{2}\right)\cdot\left(2-\sqrt{2}\right)}{\left(2+\sqrt{2}\right)\cdot\left(2-\sqrt{2}\right)} = \frac{2-\sqrt{2}-2\sqrt{2}+2}{4-2} = \frac{4-3\sqrt{2}}{2}$ $(= 2-1{,}5\sqrt{2}\,)$

i) $\frac{\sqrt{8}}{2-\sqrt{2}} = \frac{\sqrt{8}\cdot\left(2+\sqrt{2}\right)}{\left(2-\sqrt{2}\right)\cdot\left(2+\sqrt{2}\right)} = \frac{4\sqrt{2}+4}{4-2} = \frac{2\cdot(2\sqrt{2}+2)}{2} = 2\sqrt{2}+2$

k) $\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{6}\cdot\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)} = \frac{3\sqrt{2}+2\sqrt{3}}{3-2} = 3\sqrt{2}+2\sqrt{3}$

m) $\frac{\sqrt{8}-\sqrt{6}}{\sqrt{12}} = \frac{\left(\sqrt{8}-\sqrt{6}\right)\cdot\sqrt{3}}{\sqrt{12}\cdot\sqrt{3}} = \frac{2\sqrt{6}-3\sqrt{2}}{2\cdot 3} = \frac{2\sqrt{6}-3\sqrt{2}}{6}$

n) $\frac{\sqrt{12}}{1+\sqrt{2}} = \frac{\sqrt{12}\cdot\left(1-\sqrt{2}\right)}{\left(1+\sqrt{2}\right)\cdot\left(1-\sqrt{2}\right)} = \frac{2\sqrt{3}-2\sqrt{6}}{1-2} = 2\sqrt{6}-2\sqrt{3}$

p) $\frac{2+\sqrt{2}}{\left(1-\sqrt{2}\right)^2} = \frac{2+\sqrt{2}}{1-2\sqrt{2}+2} = \frac{\left(2+\sqrt{2}\right)\cdot\left(3+2\sqrt{2}\right)}{\left(3-2\sqrt{2}\right)\cdot\left(3+2\sqrt{2}\right)} = \frac{6+4\sqrt{2}+3\sqrt{2}+4}{9-4\cdot 2} = 10+7\sqrt{2}$

q) $\frac{1-\sqrt{2}}{\left(1+\sqrt{2}\right)^2} = \frac{1-\sqrt{2}}{1+2\sqrt{2}+2} = \frac{\left(1-\sqrt{2}\right)\cdot\left(3-2\sqrt{2}\right)}{\left(3+2\sqrt{2}\right)\cdot\left(3-2\sqrt{2}\right)} = \frac{3-2\sqrt{2}-3\sqrt{2}+4}{9-4\cdot 2} = 7-5\sqrt{2}$

r) $\frac{\sqrt{12}}{\left(1-\sqrt{3}\right)^2} = \frac{2\sqrt{3}}{1-2\cdot\sqrt{3}+3} = \frac{2\sqrt{3}}{4-2\sqrt{3}} = \frac{2\sqrt{3}\cdot\left(4+2\sqrt{3}\right)}{\left(4-2\sqrt{3}\right)\cdot\left(4+2\sqrt{3}\right)} = \frac{8\sqrt{3}+4\cdot 3}{16-4\cdot 3} =$

$\frac{4\cdot\left(2\sqrt{3}+3\right)}{4} = 2\sqrt{3}+3$

5. a) $2x^2-3=4 \Leftrightarrow 2x^2=7 \Leftrightarrow x^2=\frac{7}{2} \Leftrightarrow x_{1/2}=\pm\sqrt{\frac{7\cdot 2}{2\cdot 2}}=\pm\frac{\sqrt{14}}{2}$

b) $0{,}5x^2+2=3 \Leftrightarrow 0{,}5x^2=1 \Leftrightarrow x^2=2 \Leftrightarrow x_{1/2}=\pm\sqrt{2}$

c) $2\cdot(x^2-1)=5 \Leftrightarrow x^2-1=2{,}5 \Leftrightarrow x^2=3{,}5 \Leftrightarrow x_{1/2}=\pm\sqrt{\frac{7\cdot 2}{2\cdot 2}}=\pm\frac{\sqrt{14}}{2}$

d) $(x-3)\cdot(x+3)=1 \Leftrightarrow x^2-9=1 \Leftrightarrow x^2=10 \Leftrightarrow x_{1/2}=\pm\sqrt{10}$