Q12 * Astrophysik * Lösungen zu Aufgaben aus dem Lehrbuch

S. 56 / 3 $a = 1{,}08\,AE \quad \varepsilon = 0{,}83$

Aphel $x_A = a + e = a + \varepsilon a = (1+\varepsilon)a = 1{,}83 \cdot 1{,}08\,AE = 1{,}98\,AE$

Perihel $x_p = a - e = (1-\varepsilon)a = 0{,}17 \cdot 1{,}08\,AE = 0{,}18\,AE$

Ikarus kommt der Sonne sehr nahe

S. 59/2 $a_{Venus} = 0{,}72\,AE \quad T_V = 0{,}62\,a \quad T_m = 1{,}9\,a$

$$\frac{a_m^3}{T_m^2} = \frac{a_v^3}{T_v^2} \Rightarrow a_m = a_v \cdot \sqrt[3]{\left(\frac{T_m}{T_v}\right)^2} = 0{,}72\,AE \cdot \sqrt[3]{\left(\frac{1{,}9}{0{,}62}\right)^2} = 1{,}5\,AE$$

S. 59/3 im Perihel bzw. Aphel gilt $\vec{v} \perp \vec{r} \Rightarrow \Delta x_A r_A = \Delta x_p r_p \Rightarrow$

$$\Rightarrow v_A r_A = \frac{\Delta x_A}{\Delta t} \cdot r_A = \frac{\Delta x_p}{\Delta t} r_p = v_p r_p$$

S. 59/4 Halley $x_p = 0{,}587\,AE\,; \quad T_H = 76{,}1\,a$

$$\frac{a_H^3}{T_H^2} = \frac{(1AE)^3}{1a} \Rightarrow a_H = 1AE \cdot \sqrt[3]{\frac{76{,}1^2}{1^2}} = 17{,}96\,AE$$

$$x_p = a - \varepsilon a = a(1-\varepsilon) \Rightarrow \varepsilon = 1 - \frac{x_p}{a} = 1 - \frac{0{,}587}{17{,}96} = 0{,}967$$

$$x_{Aphel} = a + \varepsilon a = 17{,}96\,AE\,(1 + 0{,}967) = 35{,}3\,AE$$

S. 59/7

$$\frac{T^2}{(2AE)^3} = \frac{1a}{(1AE)^3} \Rightarrow T^2 = 8 \cdot 1a \Rightarrow T = \sqrt{8}\,a = 2{,}8\,a$$

S. 61/8 $g(\text{in der Höhe } h) = a \qquad ma = G\frac{mM_E}{(R_E+h)^2}$ und $mg = G\frac{mM_E}{R_E^2}$

$$\Rightarrow \frac{a}{g} = \frac{ma}{mg} = \frac{R_E^2}{(R_E+h)^2} \Rightarrow a = \left(\frac{R_E}{R_E+h}\right)^2 \cdot g$$

$R_E = 6370\,km$

h	1 km	10 km	100 km	1000 km
$a:g$	1,000	0,997	0,969	0,747

Für $h = 40\,km$ gilt $a = 0{,}9875... \approx 1{,}0\%$

S. 61/9

$$m\omega^2 r = G\frac{mM_J}{r^2} \Rightarrow M_J = \omega^2 \cdot \frac{r^3}{G} = \frac{4\pi^2}{T^2} \cdot \frac{r^3}{G} = \frac{4\pi^2 \cdot (671{,}4 \cdot 10^6\,m)^3 \cdot kg\,s^2}{(3d\,13h\,13min\,42s)^2 \cdot 6{,}672 \cdot 10^{-11}\,m^3}$$

$$M_J = 1{,}9 \cdot 10^{27}\,kg = \frac{1{,}9 \cdot 10^{27}}{6{,}0 \cdot 10^{24}} \cdot M_E = 3{,}2 \cdot 10^2\,M_E$$

S. 61/10 $T = 24h \qquad m\omega^2 r = G\frac{mM}{r^2} \Rightarrow r^3 = \frac{GMT^2}{4\pi^2} = \frac{6{,}67 \cdot 10^{-11} \cdot 6{,}0 \cdot 10^{24} \cdot (24 \cdot 3600)^2}{4\pi^2}\,m^3$

$$r = 42{,}3 \cdot 10^3\,km \Rightarrow h = r - R_E = (42{,}3 \cdot 10^3 - 6370)\,km = 35{,}9 \cdot 10^3\,km$$

$$v = \frac{2\pi r}{T} = \frac{2\pi \cdot 42{,}3 \cdot 10^3\,km}{24 \cdot 3600\,s} = 3{,}1\,\frac{km}{s}$$

S. 61/11 Aus Tabellen: $M_E = 5{,}97 \cdot 10^{24}\,kg\,; \quad M_m = 7{,}35 \cdot 10^{22}\,kg\,; \quad x_{Erde-Mond} = 60 \cdot R_E$

$$G\frac{mM_E}{x^2} = F_G(m) = G\frac{mM_m}{(60R_E - x)^2} \Rightarrow (60R_E - x)^2 = \frac{M_m}{M_E} \cdot x^2 \Rightarrow$$

$$60R_E - x = \sqrt{\frac{M_m}{M_E}} \cdot x \Rightarrow 60R_E = \left(1 + \sqrt{\frac{M_m}{M_E}}\right)x \Rightarrow x = \frac{60R_E}{1 + \sqrt{M_m/M_E}} = 54\,R_E$$

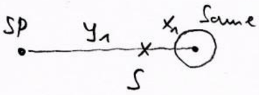
S. 63/12 $\quad m_J = 1{,}9 \cdot 10^{27}\,kg \qquad m_S = 5{,}69 \cdot 10^{26}\,kg \qquad m_\odot = 1{,}99 \cdot 10^{30}\,kg$

$r_J = 7{,}78 \cdot 10^{8}\,km \qquad r_S = 1{,}43 \cdot 10^{9}\,km \qquad R_\odot = 6{,}96 \cdot 10^{5}\,km$

$\frac{x}{y} = \frac{m_S}{m_J} = 0{,}2995 \; ; \quad x + y = r_S - r_J = 6{,}52 \cdot 10^{8}\,km$

$\Rightarrow 1{,}2995\,y = 6{,}52 \cdot 10^{8}\,km \Rightarrow x = 1{,}5 \cdot 10^{8}\,km$

SP $\hat{=}$ Schwerpunkt Saturn/Jupiter



S: Schwerpunkt von Saturn, Jupiter und Sonne!

$\frac{x_1}{y_1} = \frac{m_S + m_J}{m_\odot} = 1{,}24 \cdot 10^{-3}\,km \qquad \frac{y_1}{x_1} = 806$

$x_1 + y_1 = x + r_J \Rightarrow x_1 + y_1 = (7{,}78 + 1{,}5) \cdot 10^{8}\,km$

$\Rightarrow 807\,x_1 = 9{,}28 \cdot 10^{8}\,km \Rightarrow x_1 = 1{,}15 \cdot 10^{6}\,km$

$x_1 = \frac{1{,}15 \cdot 10^{6}}{6{,}96 \cdot 10^{5}} \cdot R_\odot = 1{,}65 \cdot R_\odot$

S. 63/13 a) $\quad \frac{T^2}{r^3} = \frac{4\pi^2}{G(m_p + m_c)} \Rightarrow m_p + m_c = \frac{4\pi^2 \cdot (19\,400 \cdot 10^3\,m)^3}{6{,}67 \cdot 10^{-11}\,\frac{m^3}{kg\,s^2} \cdot (6{,}3 \cdot 24 \cdot 3600\,s)^2} = 1{,}46 \cdot 10^{22}\,kg$

13 b) $\quad m \sim V \sim r^3 \Rightarrow \frac{m_p}{m_c} = \left(\frac{r_p}{r_c}\right)^3 = \left(\frac{2200}{1160}\right)^3 = 6{,}82 \Rightarrow m_{ges} = 7{,}82\,m_c$

$m_c = 0{,}13\,m_{ges} = 13\,\%$ von m_{ges} ; $m_p = 87\,\%$ von m_{ges}

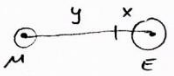
13 c) $\quad m_p = 0{,}87 \cdot (m_p + m_c) = 0{,}87 \cdot 1{,}46 \cdot 10^{22}\,kg = 1{,}3 \cdot 10^{22}\,kg$

S. 63/14

a) $\quad \frac{T_{sid}^2}{r^3} = \frac{4\pi^2}{G(m_M + m_E)} \Rightarrow r^3 = \frac{T_{sid}^2 \cdot G \cdot \frac{82}{81} \cdot m_E}{4\pi^2} \Rightarrow$

$r = \sqrt[3]{\frac{(27{,}3 \cdot 24 \cdot 3600\,s)^2 \cdot 6{,}67 \cdot 10^{-11}\,\frac{m^3}{kg\,s^2} \cdot 82 \cdot 5{,}97 \cdot 10^{24}\,kg}{81 \cdot 4 \cdot \pi^2}} = 3{,}84 \cdot 10^{8}\,m$

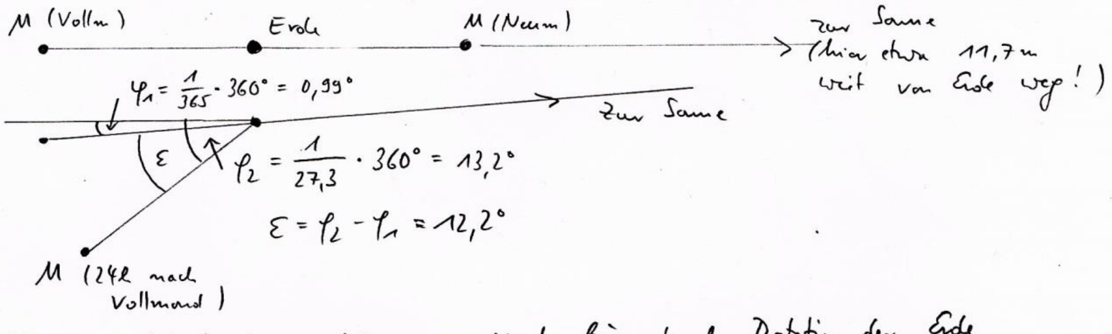
b)



$\frac{x}{y} = \frac{m_M}{m_E} = \frac{1}{81}$ und $x + y = 3{,}84 \cdot 10^{8}\,m \Rightarrow 82\,x = 3{,}84 \cdot 10^{8}\,m$

$\Rightarrow x = 4{,}7 \cdot 10^{6}\,m \approx 0{,}74 \cdot R_{Erde} \qquad (R_{Erde} = 6370\,km)$

c)



Flutberg behält Lage relativ zum Mond bei; durch Rotation der Erde um Achse (in 24h) wandert der Flutberg relativ zur Erde, allerdings dreht sich auch der Mond noch um $\varepsilon = 12{,}2°$ weiter!

Mehrbetrag: $\quad \Delta t = \frac{12{,}2°}{360°} \cdot 24\,h = 48{,}8\,min$

d) $\quad F_G \sim \frac{M}{r^2} \Rightarrow F_S : F_E = \frac{M_\odot}{(1\,AE)^2} : \frac{M_E}{(384 \cdot 10^3\,km)^2} = \frac{2{,}0 \cdot 10^{30}\,kg}{6{,}0 \cdot 10^{24}\,kg} \cdot \frac{(384 \cdot 10^3\,km)^2}{(150 \cdot 10^6\,km)^2} =$

$= 2{,}2 = 2{,}2 : 1$